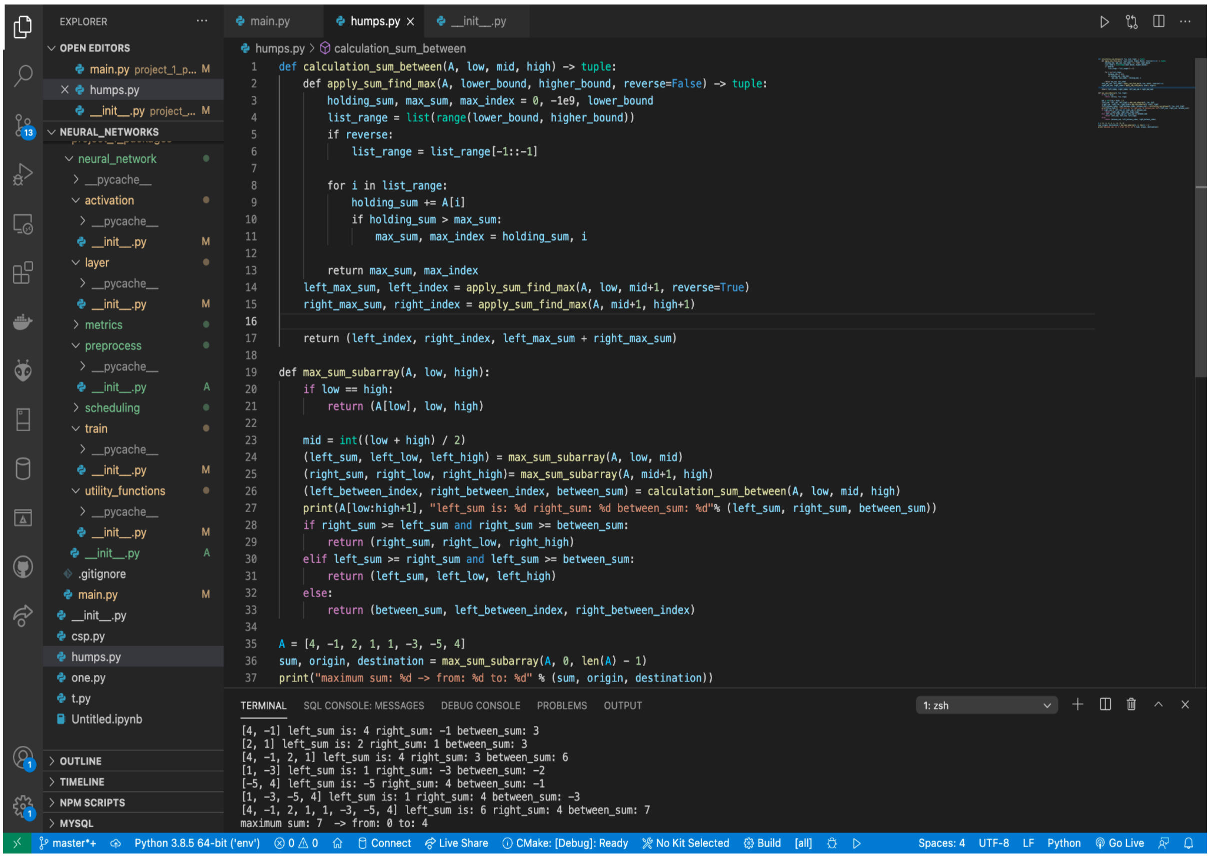
Algorithm Design 3992
April 2021
Instructor : Dr.Shafiei

Question 1.



Question 2.

The first matrices are

$$
\begin{aligned}
S_1 &= 6 & S_6 &= 8\\
S_2 &= 4 & S_7 &= -2\\
S_3 &= 12 & S_8 &= 6\\
S_4 &= -2 & S_9 &= -6\\
S_5 &= 6 & S_{10} &= 14.
\end{aligned}
$$

The products are

$$
\begin{aligned}
P_1 &= 1 \cdot 6 = 6\\
P_2 &= 4 \cdot 2 = 8\\
P_3 &= 6 \cdot 12 = 72\\
P_4 &= -2 \cdot 5 = -10\\
P_5 &= 6 \cdot 8 = 48\\
P_6 &= -2 \cdot 6 = -12\\
P_7 &= -6 \cdot 14 = -84.
\end{aligned}
$$

The four matrices are

$$
\begin{aligned}
C_{11} &= 48 + (-10) - 8 + (-12) = 18\\
C_{12} &= 6 + 8 = 14\\
C_{21} &= 72 + (-10) = 62\\
C_{22} &= 48 + 6 - 72 - (-84) = 66.
\end{aligned}
$$

The result is

$$
\begin{pmatrix} 18 & 14 \\ 62 & 66 \end{pmatrix}.
$$

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Question 3.

- Master Theory :
  - T(n) = a . T(n / b) + O(n ^ d)

a. $T(n) = 3T(n / 9) + \sqrt{n}$
A = 3 , b = 9 , d = ½ → a = b ^ d → $O(n^{1/2} . log(n))$

b. $T(n) = T(n - 4) + n$
$\rightarrow T(n) = n + (n - 4) + (n - 8) + ... + 4 + T(0)$
$T(n) = \frac{n}{8}[2a + (\frac{n}{4} - 1) . 4] + T(0) = \frac{n}{8}[n + 4] + T(0) = \frac{n^2}{8} + \frac{n}{2} + T(0) \in$

c. $T(n) = 6T(n / 4) + n^2$
A = 6 , b = 4 , d = 2 → a < b ^ d → O(n ^ 2)

d. $T(n) = 5T(n / 2) + n^2$
A = 5 , b = 2 , d = 2 → a > b ^ d → $O(n^{Log\ 5\ base\ 2})$

Question 4.

$T(n) = 2T(n / 3) + n \ for\ n \geq 5$
A = 2 , b = 3 , d = 1 → b ^ d > a → O(n)

Question 5.

We can get to correct answer by testing the options and also we can solve it by writing characteristic equation:

$x^2 - 5x + 6 = 0 \rightarrow x = 2, 3 \rightarrow g(n) = \alpha 1\ (3)\ n + \alpha 2\ (2)$

Question 6.

Step 1 : moving n-1 discs from A to B using C
Step 2: moving one disc from A to C
Step 3: moving n-1 discs from B to A using C
Step 4: moving one disc from C to B
Step 5: moving n-1 disc from A to B using C
In Conclusion :

$$T(n - 1) + 1 + T(n - 1) + 1 + T(n - 1) = 3T(n - 1) + 2$$